UNIVERSITY OF RIJEKA
DEPARTMENT OF MATHEMATICS

Department of Mathematics
University of Rijeka

Discrete Mathematics and its Applications

Europska unija
"Zajedno do fondova EU"
EUROPSKI STRUKTURNI I INVESTICIJSKI FONDOVI
UČINKOVITI LJUDSKI POTENCIJALI

Projekt je sufinancirala Europska unija iz Europskog socijalnog fonda

Projekt „Strateška internacionalizacija diplomskih studija matematike i biotehnologije – OPTILIFE“ je sufinancirala Europska unija iz Europskog socijalnog fonda.

Za više informacija o EU fondovima posjetite www.strukturnifondovi.hr

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For more information on EU fonds visit www.strukturnifondovi.hr

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SADRŽAJ

CONTENT

PROJEKT OPTILIFE

Projektom „Strateška internacionalizacija diplomskih studija matematike i biotehnologije – OPTILIFE“ značajno će se ojačati kapaciteti Sveučilišta u Rijeci za internacionalizaciju te će se pridonijeti kvaliteti i relevantnosti visokog obrazovanja.

Navedeno će se postići razvojem novog diplomskog studija „Biotehnološka istraživanja znanosti o životu“ Odjela za biotehnologiju te razvojem novog modula „Optimizacija” u sklopu diplomskog studija „Diskretna matematika i primjene“ Odjela za matematiku, kao i izvedbom dijela navedenih studija. Oba studija izvodit će se na engleskom jeziku, a temeljeni su na novim zahtjevima tržišta rada.

Povećanjem broja studijskih programa na stranim jezicima uz jačanje kompetencija studenata i osoblja visokog učilišta želi se dodatno potaknuti znanstvena izvrsnost i mobilnost na Sveučilištu u Rijeci. Interdisciplinarnim pristupom i fokusom na vještine potrebne za istraživanje, studentima ovih studija bit će omogućena kvalitetna priprema za buduću istraživačku karijeru te će biti kompetitivni pri prijavljivanju na doktorske studije kao i prilikom zapošljavanja.

Korisnik projekta je Sveučilište u Rijeci (Odjel za matematiku i Odjel za biotehnologiju), dok su partneri projekta Filozofski fakultet u Rijeci i Login d.o.o.

Projekt je započeo 12. listopada 2018. te traje do 12. listopada 2021. godine.

Ukupna vrijednost projekta iznosi 1.799.346,08 kuna, od čega je 85 % osigurano iz Europskog socijalnog fonda, a 15 % iz Državnog proračuna Republike Hrvatske.

OPTILIFE PROJECT

The project „Strategic Internationalisation of Graduate Studies in Mathematics and Biotechnology – OPTILIFE" will strengthen the capacity of the University of Rijeka for internationalisation and enhance the quality and relevance of higher education at the University.

The project will enable the development of a new graduate programme of the Department of Biotechnology "Biotechnology for the Life Sciences" and the development of a new module "Optimisation" as part of graduate programme of the Department of Mathematics "Discrete Mathematics and Its Applications", as well as partial implementation of the above mentioned studies. Both studies will be carried out in the English language and are based on new demands of the labour market.

By enhancing the number of study programmes in the English language and strengthening the competencies of students and staff, the University of Rijeka wishes to stimulate scientific excellence and mobility. With an interdisciplinary approach and a focus on skills required for research, these two studies will provide quality preparation for students wishing to pursue a career in research and will provide a competitive advantage for students when looking for employment opportunities.

The project beneficiary is the University of Rijeka (Department of Mathematics and Department of Biotechnology), while the project partners are Faculty of Humanities and Social Sciences in Rijeka and Login Ltd.

The project started on October 12, 2018 and will last until October 12, 2021.

The total value of the project is 1,799,346.08 HRK (approx. 242,000 EUR), with 85 % of funding secured from the European Social Fund and 15 % from the State Budget of the Republic of Croatia.

O Hrvatskoj

Suvremena Hrvatska, nasljednica je hrvatskih srednjovjekovnih kneževina iz 9. st. utemeljenih u marševima Karolinškog Carstva, potom i Hrvatskoga Kraljevstva, koje je nastalo 925. godine krunidbom kralja Tomislava. Republika Hrvatska članica je Europske unije od 2013. Njezina površina obuhvaća 5 694 km², a Jadranska je obala treća po dužini na Mediteranu. Duljina hrvatske obale je 1 777 km, a duljina obale 1 185 otoka je 4 058 km. Samo 47 otoka je naseljeno. Prema popisu stanovništva iz 2011. godine Hrvatska ima 4,29 milijuna stanovnika, a njih 700 000 živi u glavnom gradu Zagrebu.

O gradu Rijeci

Rijeka, treći najveći grad u Hrvatskoj, ima oko 130 000 stanovnika. Nalazi se u Primorsko-goranskoj županiji u Kvarnerskom zaljevu, upravo tamo gdje se u Jadransko more ulijeva krška rijeka Rječina, duga 19 km. Rijeka ima ugodnu mediteransku klimu s toplim ljetom i relativno blagom zimom. Ima bogatu povijest koja je ostavila snažne tragove u lokalnom životu i poznata je po svom karnevalu, koji se održava svake godine između kraja siječnja i početka ožujka. Rijeka je odabrana kao Europska prijestolnica kulture za 2020. zajedno s gradom Galway u Irskoj.

About Croatia

Contemporary Croatia is the successor of the 9th-century Croatian medieval principalities established in the marches of the Carolingian Empire, followed by the Kingdom of Croatia, founded in 925 by King Tomislav. The Republic of Croatia is a member of the European Union since 2013. It covers an area of 56,594 square km and its Adriatic shoreline is the third longest in the Mediterranean. The length of the coastline of Croatian mainland is 1,777 km, and the coastline length of its 1,185 islands is 4,058 km. Out of the total number of islands, only 47 islands are inhabited. According to the 2011 census, Croatia has 4.29 million inhabitants, and 700,000 of them lives in its capital city Zagreb.

About the city of Rijeka

Rijeka, the third largest city in Croatia, has approximately 130,000 inhabitants. It is located in Primorje-Gorski Kotar County on Kvarner Bay, an inlet of the Adriatic Sea. Just where a karst river Rječina, 19 km long, flows into it. Rijeka has a pleasant Mediterranean climate with warm summers and relatively mild winters. It has a rich history that has left strong traces in the local life, and it is known for its carnival, held every year between late January and early March. Rijeka was selected as European Capital of Culture for 2020 along with Galway, Ireland.

croatia.eu
www.rijeka.hr

Sveučilište u Rijeci

Sveučilište u Rijeci osnovano je 1973. godine, a danas je ono suvremeno europsko sveučilište i centar izvrsnosti unutar regije i šire. 2018. godine proslavilo je 45. obljetnicu te obilježilo 385 godina visokoškolskog obrazovanja u Rijeci. S ukupno 11 fakulteta i 4 odjela predstavlja istraživačko i znanstveno-obrazovno sveučilište odgovorno za društveni i gospodarski razvoj zajednice, grada Rijeke i regije.

Riječko je sveučilište društveno odgovorno i otvoreno sveučilište – otvoreno stranim studentima, nastavnome osoblju, istraživačima, ali i suvremenim trendovima u visokome školstvu. Vizija Sveučilišta u Rijeci je ulazak među 500 najboljih europskih sveučilišta, stoga se ono opredijelilo za dinamičan razvoj koji sustavno i organizirano potiče mobilnost i razvijanje istraživačkih i umjetničkih karijera te omogućuje izražavanje talenta i poduzetničke energije svakog pojedinca.

University of Rijeka

The University of Rijeka (UNIRI) is a modern European university and a center of excellence in the region and beyond. In 2018 Rijeka celebrated the 385th anniversary of higher education in Rijeka and the 45th anniversary of University of Rijeka. It consists of 17 constituents, encompasses 172 studies, and has approximately 17,000 students. In alignment with European standards, Croatia's higher education system has adopted the best features of the Bologna Process, and the University of Rijeka has been recognized as a student-centered university which provides high student standard.

There are two student dormitories in Rijeka, one of which is a new one, "Trsat" on Kampus and opened in July 2016.


5th Croatian Mathematical
Congress
June 18 – 21, 2012
Rijeka, Croatia

UNIVERSITY OF RIJEKA
DEPARTMENT OF MATHEMATICS

Odjel za matematiku Sveučilišta u Rijeci

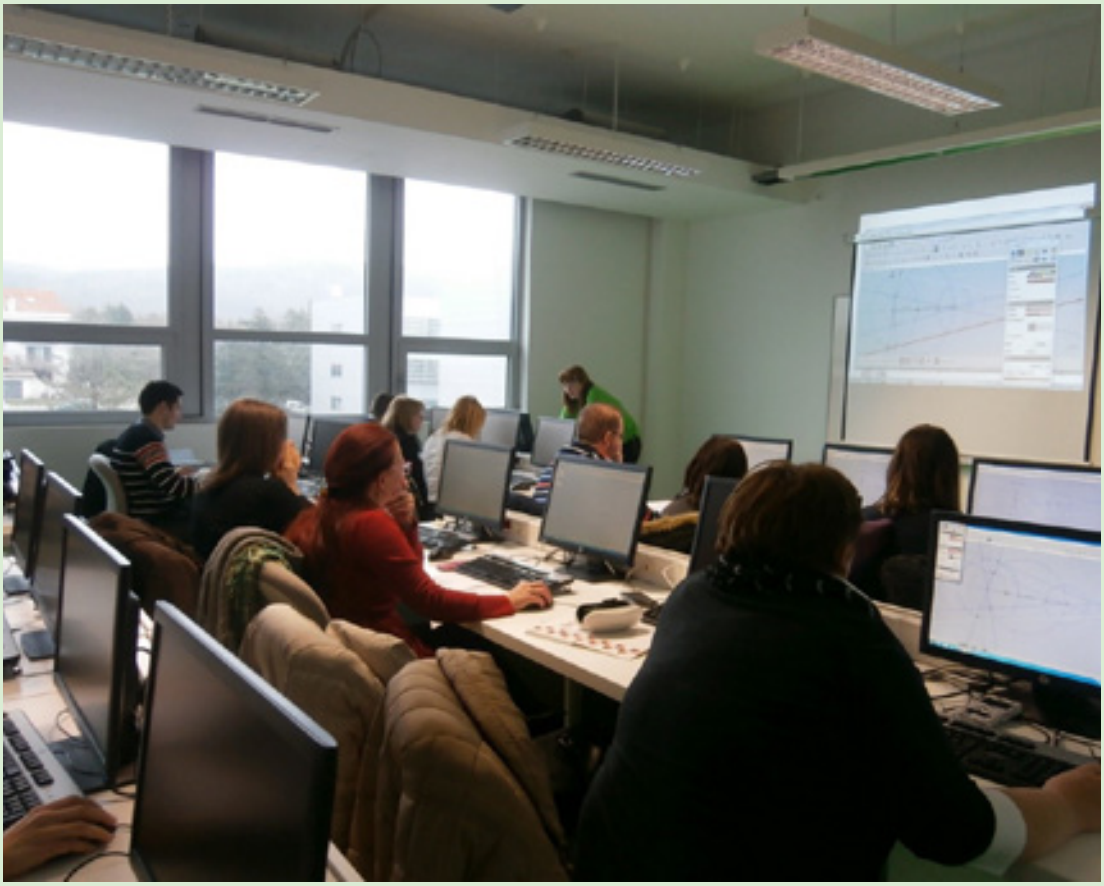
Odjel za matematiku Sveučilišta u Rijeci je znanstveno-nastavna sastavnica Sveučilišta u Rijeci koja razvija znanstveni i stručni rad u znanstvenom polju matematike, sudjeluje u organiziranju i izvedbi studijskih programa i studija, organizira studij iz znanstvenog polja matematika te vodi brigu o razvoju kadrova iz toga znanstvenog polja na Sveučilištu u Rijeci.

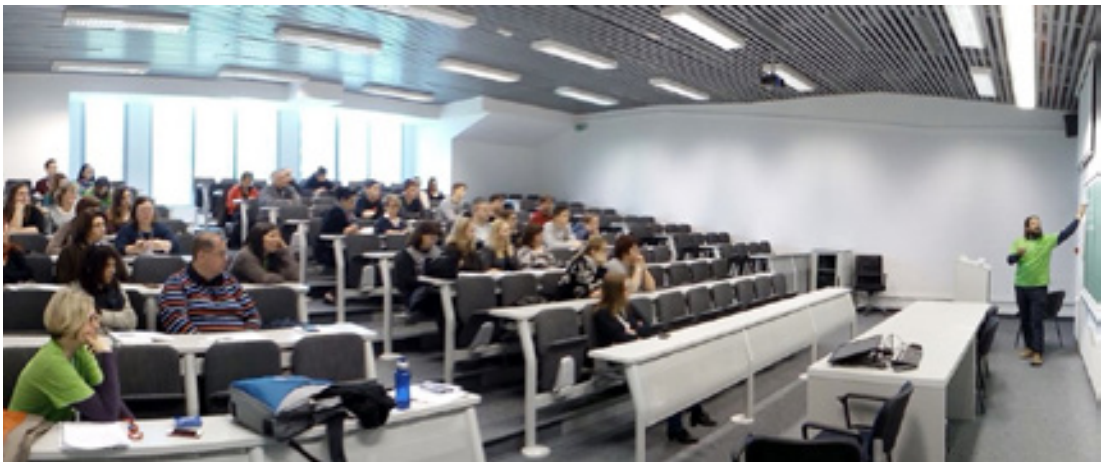
MISIJA ODJELA ZA MATEMATIKU

Odjel za matematiku Sveučilišta u Rijeci je znanstveno-nastavna sastavnica Sveučilišta koja razvija znanstveni i stručni rad u znanstvenom polju matematika i vodi brigu o razvoju kadrova iz znanstvenog polja matematika na Sveučilištu u Rijeci. Odjel za matematiku organizira i izvodi studije iz svog područja, te sudjeluje u organiziranju i izvedbi studijskih programa na drugim znanstveno nastavnim sastavnicama Sveučilišta. Odjel za matematiku pridonosi razvoju Sveučilišta i društva težeći nacionalno i međunarodno prepoznatoj izvrsnost u znanstvenoj i nastavnoj djelatnosti.

VIZIJA ODJELA ZA MATEMATIKU

Odjel za matematiku Sveučilišta u Rijeci želi se strateški pozicionirati kao međunarodno prepoznata i priznata znanstveno-nastavna institucija i aktivna sastavnica Hrvatskog i Europskog visokoobrazovnog i istraživačkog prostora koja obrazuje međunarodno priznate kvalitetne i kompetentne stručnjake i provodi međunarodno prepoznata znanstvena istraživanja.





Department of Mathematics

The Department of Mathematics, University of Rijeka, is one of the constituents of University of Rijeka which does research and professional work in the field of mathematics, organizes study programmes in mathematics and assures the development of the personnel in mathematics at the University of Rijeka. Since 2012, it is situated in the new Trstat Campus.

The establishment of the Department of Mathematics and Applied Physics at the Higher Vocational School of Pedagogy on February 20th, 1961, whose constituent part was the Chair of Mathematics, can be considered as the beginning of higher education in the field of mathematics in Rijeka. In the academic year 1964/1965 the four-year study programme of mathematics and physics was launched.

The Department of Mathematics, University of Rijeka, was founded as a constituent of the Univertsity of Rijeka on December 17th, 2007, by the decision of the establishment of the Department of Mathematics, University of Rijeka, which was passed by the University Senate.

Since its establishment, the Department of Mathematics was developing in accordance with its vision, that is, the strategic positioning of the Department as an internationally recognized and accepted scientific and educational institution, as well as an active component of Croatian and European higher education and research, educating internationally recognized, quality and capable professionals and conducting internationally recognized research.

Ustroj Odjela za matematiku Sveučilišta u Rijeci

Zavodi Odjela za matematiku Sveučilišta u Rijeci ustrojeni su prvenstveno na temelju povezanosti i srodnosti znanstveno-istraživačkog rada.

— Zavod za algebru i teoriju brojeva
— Zavod za diskretnu matematiku
— Zavod za matematičku analizu

Zavod za algebru i teoriju brojeva

Znanstveno istraživanje članova Zavoda za algebru i teoriju brojeva u područjima je algebre, teorije brojeva, matematičke analize, geometrije i topologije te računarstva, a obuhvaća sljedeće smjerove koji se međusobno dopunjuju i isprepliću:

— afine Liejeve algebre i algebre verteks operatora,
— Diofantske jednadžbe i polinomne varijante Diofantova problema,
— teorije dimenzije, rezolucioni teoremi,
— geometrijska teorija grupa,
— formalni modeli sustava suradnje, analiza sigurnosnih protokola.

Zavod za diskretnu matematiku

Znanstvena djelatnost članova Zavoda za diskretnu matematiku je pretežno u području kombinatorne i diskretne matematike, a posebno se istražuju problemi iz sljedećih, međusobno usko povezanih, područja:

— teorija dizajna,
— teorija grafova,
— teorija kodiranja.

Zavod za matematičku analizu

Zavod okuplja djelatnike koji rade u sličnim, istim ili kompatibilnim znanstvenim područjima utemeljenim na matematičkoj analizi. U proteklih devet godina znanstveno-istraživački rad djelatnika Zavoda odvijao se u sljedećim područjima matematike:

— funkcionalna analiza,
— dinamički sustavi,
— numerička matematika i primjene,
— diferencijalna geometrija,
— obične i parcijalne diferencijalne jednadžbe i mehanika kontinuuma,

Structure of the Department of Mathematics, University of Rijeka

The Division of Algebra and Number Theory

The Division of Algebra and Number Theory is a part of the Department of Mathematics, University of Rijeka, since January 2011. It was founded by dividing up the former Division of Algebra and Discrete Mathematics. Scientific activities of the members of the Division pertain to the areas of algebra, number theory, mathematical analysis, geometry, topology and computing. The Division members' area of research interests covers the following directions that complement each other and interlock:

- Affine Lie algebras and vertex operator algebras,
- Diophantine equations and polynomial variants of the problem of Diophantus,
- Dimension theories, resolution theorems,
- Geometric group theory,
- Formal models of collaboration systems, analysis of the security protocols.

Division of Discrete Mathematics

The Division of Discrete Mathematics of the Department of Mathematics, University of Rijeka, was founded in January 2011, when the Division of Algebra and Discrete Mathematics, established at the beginning of 2009, was divided into two divisions: the Division of Algebra and Number theory and the Division of Discrete Mathematics. The scientific activity of the members of the Division is mainly in the field of combinatorics and discrete mathematics, focusing on the problems in the following closely related areas:

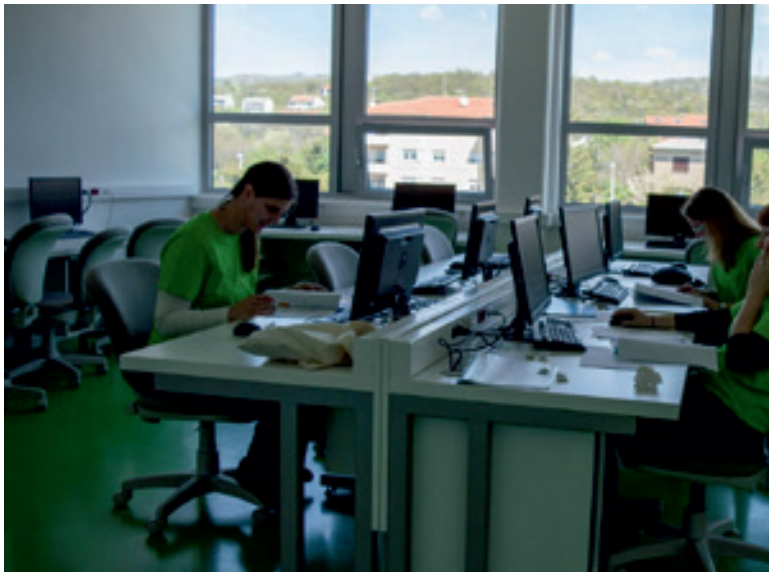
- Design theory,
- Graph theory,
- Coding theory.

The members of the Division of Discrete Mathematics of the Department of Mathematics form the biggest research group in discrete mathematics in Croatia.

The Division of Mathematical Analysis

The Division of Mathematical Analysis of the Department of Mathematics of the University of Rijeka was founded on January 28, 2009. The Division gathers staff members that work in similar, same or compatible scientific areas based on Mathematical Analysis. The members of the Division conduct research in the following fields of mathematics:

- Functional Analysis,
- Dynamical Systems,
- Numerical Analysis and Applications,
- Differential Geometry,
- Probability Theory and Statistics.




CAFFE BAR PORTULA

ODJEL ZA MATEMATIKU SVEUČILIŠTA U RIJECI NAJBROJNIJA ISTRAŽIVAČKA GRUPA BAVI SE DISKR

VEKARIZA

Matematička znanja sve više traže banke – prof. dr. Sanja Rukavina i prof. dr. Dean Crnković

Iznimno primjenjiva grana matematike čiji značaj raste

Diskretna matematika proučava diskretne, odnosno skupove koji nisu kontinuirani. Obuhvaća kombinatoriku, teoriju grafova, teoriju kodiranja i kriptografiju, odnosno šifriranje, tumače prof. dr. Dean Crnković, pročelnik Odjela, i prof. dr. Sanja Rukavina, zamjenica pročelnika

Aneli DRAGOJEVIĆ MIJATOVIĆ

Na Odjelu za matematiku Sveučilišta u Rijeci provode se znanstvena istraživanja iz više grana matematike, a najbrojnija istraživačka grupa je ona koja se bavi diskretnom matematikom. Diskretna matematika je grana matematike koja proučava diskretne skupove, odnosno one skupove koji nisu kontinuirani. Diskretni skupovi su, primjerice, skup prirodnih brojeva, skup cijelih brojeva i konačni skupovi. U suvremenu diskretnu matematiku ubrajaju se, između ostaloga, kombinatorika, teorija grafova, teorija kodiranja (problemi prijenosa podataka kroz kanale sa smetnjama) i kriptografija, odnosno šifriranje (problemi zaštite tajnosti podataka). O diskretnoj matematici, njenim primjenama, te aktivnostima Odjela za matematiku riječkog sveučilišta, za »Vox Academiae« govore pročelnik Odjela prof. dr. Dean Crnković, te zamjenica pročelnika Odjela prof. dr. Sanja Rukavina.

– Diskretna matematika je dio matematike koji se bavi proučavanjem diskretnih skupova. Diskretni skupovi su oni koji nisu kontinuirani. Kontinuiran je primjerice skup realnih brojeva: kada crtamo pravac povlačimo liniju u kontinuitetu, ne dižemo olovku s papira. Diskretan skup je skup prirodnih brojeva, skup cijelih brojeva. On je pak skokovit, točkast, i kada bismo dakle na pravcu označavali elemente skupa cijelih brojeva, stavljali bismo točkice a ne vukli crtu, opisuje Crnković, odgovarajući na našu zamolbu da pokuša što jednostavnije pojasniti pojam diskretne matematike. Nadalje, diskretna matematika ima i sve veću primjenu u računarstvu, zbog čega njezin značaj iz dana u dan raste. To me govori u prilog i činjenica da su istraživanja iz područja diskretne matematike na Odjelu za matematiku rezultirala organizacijom znanstvenog skupa »Information Security and Related Combinatorics«.

NATO financirao skup

Ovaj znanstveni skup organiziran je 2010. godine u Opatiji u okviru NATO-vog programa Science for Peace and Security, u formi NATO Advanced Study Institute. Na skupu je sudjelovalo više od 80 sudionika iz 12 država.

– Budući da ima veze sa sigurnošću prijenosa informacija, onda je očekivano da će institucija poput NATO-a podupriječi istraživanja iz određenih područja diskretne matematike. Kodiranje je prijenos informacija, i kada, primjerice, prenosite informacija s nekog satelita - a često se spominje primjer misije koja ide na Mars te treba poslati neku sliku na Zemlju - naravno da tu dolazi do puno smetnji i morate za taj prijenos imati dobar kod. Kriptografija ili šifriranje bavi se tajnošću podataka što je iznimno aktualno područje, važno sa stajališta sigurnosti općenito. Pojednostavljeno, kriptografija osigurava da vam netko ne šalje poruku umjesto onoga tko vam treba slati poruku ili da netko ne presretne poruku koju vam netko drugi šalje. Kada šaljete neki podatak, to se pretvori u niz nula i jedinica, što je vrlo mali skup od samo dva elementa. Poruku šifrirate u neki određeni niz nula i jedinica. Nula i jedan čine konačan skup od dva elementa kojima šifrirate svaku poruku. I zato je to, zbog konačnosti skupa, diskretna matematika, pojašnjava Crnković. Pitamo o kombinatorici, formalnoj logici, teoriji igara..., što su područja koja se također vežu uz diskretnu matematiku.

> *Pokretanjem ovog studija popunjena je praznina koji smo imali u visoko obrazovnom sustavu*
>
> *Prof. dr. Sanja Rukavina*

– Na Odjelu se bavimo prije svega teorijom kodiranja, teorijom dizajna, te teorijom grafova, koja je isto tako iznimno primjenjiva u stvarnom životu. Recimo, kada putujete i tražite najkraći put između dvije destinacije, zapravo koristite algoritam jer algoritam ceste i gradove pretvara u graf... Svakodnevno dakle koristimo algoritme iz teorije grafova, no kao korisnici naravno i ne idemo za tim što je pozadina toga, a pozadina je teorija grafova, odnosno opet diskretna matematika, tumači dalje Crnković.

Inače, istraživačka grupa iz područja diskretne matematike u posljednje je četiri godine svoja istraživanja provodila u okviru istraživačkog projekta »Kodovi i s njima povezane kombinatoričke strukture«, čiji je voditelj prof. dr. Dean Crnković. To je, kažu naši sugovornici, jedini projekt u području diskretne matematike financiran od strane Hrvatske zaklade za znanost. U njegove se ishode ubraja preko 40 znanstvenih članaka, publiciranih u međunarodnim znanstvenim časopisima, velik broj sudjelovanja na međunarodnim znanstvenim skupovima, usavršavanje mladih istraživača na međunarodnim ljetnim školama, kao i četiri obranjene doktorske disertacije. Uspostavljena je i suradnja s kolegama iz Slovenije koja je rezultirala bilateralnim znanstveno-istraživačkim projektom »Dizajni, kodovi, grafovi i kriptografija - interdisciplinarni pristup analizi diskretnih struktura«. Voditeljica s hrvatske strane je prof. dr. Sanja Rukavina.

Diplomski studij

Razvoj istraživanja u području diskretne matematike na Odjelu za matematiku Sveučilišta u Rijeci doveo je i do organizacije Diplomskog sveučilišnog studija Diskretna matematika i primjene, koji se na Odjelu za matematiku izvodi od akademske godine 2011./2012. To je jedini di-

Formalna i fuzzy logika

Na pitanje o vrstama logike i »pripadajućim« matematikama, Crnković odgovara da se time bave matematička logika i filozofija matematike.

– Jasno je da formalna logika radi u binarnom sustavu - nula i jedan, a postoji i logika koja proučava sva stanja između nule i jedinice (fuzzy logic, op.a.). U matematici nema prijepora u smislu koja je logika ispravna, već kažemo: uz ove pretpostavke vrijede ove tvrdnje, a uz neke druge pretpostavke vrijede drugačije tvrdnje, tumači Crnković.

ETNOM MATEMATIKOM

Diskretne strukture spojive s iracionalnim brojevima

– Diskretne strukture koje proučavamo imaju neke svoje elemente; oni jesu diskretni u smislu da se mogu jasno razlučiti, no što je zapravo sadržaj elementa, koju informaciju nosi, što je kodirano, prikazano, i što se krije iza svega toga, to može biti nešto sasvim drugo… Pojedinim točkama mogu se naime pridružiti i neke iracionalne vrijednosti. Stvar je dakle malo složenija i ne može se baš samo tako reći da čim vidiš iracionalni broj, on nema nikakve veze s diskretnom matematikom, pojašnjava profesorica Rukavina. Iracionalni brojevi su naime oni koji imaju neograničeno mnogo decimala koje se k tome ne pojavljuju u pravilnim serijama. Sami po sebi izlaze iz domene diskretne matematike. No, s njome, kako pojašnjava profesorica Rukavina, uopće nisu nespojivi.

plomski studij u području diskretne matematike u Hrvatskoj, a zbog razvijenog algoritamskog načina razmišljanja i sposobnosti brzog prilagođavanja u radu s novim informacijskim tehnologijama studenti koji završe studij visoko su zapošljivi i lako pronalaze posao.

– Kodiranje ima još i svakodnevniju primjenu, primjerice u telekomunikacijama. Podaci se prenose kada telefoniramo, kad gledamo televiziju, sve su to prijenosi podataka. Tako da se, s obzirom da je u posljednje vrijeme došlo do snažnog razvoja telekomunikacija i različitih mrežnih sustava općenito, te s obzirom da sve to postaje znatno složenije nego je bilo prije, pojavljuje i potreba za stručnjacima koji će se znati nositi s time. Pitanje je dakle kako te složene sustave riješiti na optimalan način i kako uopće sve te podatke kojih ima sve više prenijeti na siguran način. S obzirom da se ljudi na Odjelu za matematiku bave diskretnom matematikom već dulje vrijeme, odnosno ona je već dugo predmet našeg znanstvenog interesa, te s obzirom da ovakav studij nije postojao nigdje u Hrvatskoj, bilo je prirodno pokrenuti ovakav studij kod nas u Rijeci i na neki način popuniti tu prazninu koju smo imali u našem visokoobrazovnom sustavu. Pokrenuli smo stoga studij diskretne matematike i njene primjene, a želja nam je da dio koji se odnosi na primjenu još i dodatno ojačamo, kaže Rukavina.

Inače, za internacionalizaciju ovog studijskog programa dobivena su bespovratna sredstva Europskih strukturnih fondova u okviru projekta Strateška internacionalizacija diplomskih studija matematike i biotehnologije – OPTILIFE. U sklopu ovog projekta planira se u okviru Diplomskog sveučilišnog studija Diskretna matematika i primjene razviti novi izborni modul Optimizacija, pripremiti nastavne materijale za izvođenje studija na engleskom jeziku, te za prvu generaciju polaznika pokrenuti studij na engleskom jeziku.

– Vidjeli smo priliku da studij koji imamo a koji se bavi ovom iznimno aktualnom tematikom ponudimo i na engleskom jeziku, tumači Rukavina. Inače, kada govorimo o zapošljivosti, matematička znanja sve više traže i banke, pa tako njihovi studenti rade u bankama, informatičkim tvrtkama, a neki ostaju i na fakultetu usavršavati se i baviti znanstveno-istraživačkim radom.

Primjenjivo u gospodarstvu

U monografiji Odjel za matematiku Sveučilišta u Rijeci 2007.-2017. navodi se kako je »znanje stečeno na studiju Diskretna matematika i primjene vrlo primjenjivo u gospodarstvu: teorija grafova ima široku primjenu, od telekomunikacija do projektiranja cestovnih mreža, teorija kodiranja i kriptografija svakodnevno se primjenjuju u komuniciranju, optimizacija je izuzetno svrhovita u raznim poslovnim procesima, dok su dizajniranje i analiza eksperimenata nužni pri provođenju bilo kojeg eksperimenta, od proizvodnje novih lijekova do testiranja strojeva i njihovih dijelova.« Također, znanja iz područja dizajniranja eksperimenata vrlo su primjenjiva i pri ispitivanju karakteristika gotovih proizvoda, ističe se.

istockphoto

Diskretna matematika i primjene

Diskretna matematika je grana matematike koja ima brojne primjene u znanosti i gospodarstvu.

U okviru studijskog programa "Diskretna matematika i primjene" studenti će, između ostaloga, steći znanja iz teorije grafova, kriptografije, teorije kodiranja, dizajniranja eksperimenata i optimizacije.

Teorija grafova ima široku primjenu u računarstvu, kemiji, biologiji, fizici, lingvistici i društvenim znanostima, od telekomunikacija do dizajna prometnih mreža i proučavanja društvenih mreža.

Teorija kodiranja primjenjuje se u svakodnevnoj komunikaciji za prijenos podataka te za njihovo pohranjivanje, dok je kriptografija ključna za osiguravanje tajnosti podataka (npr. kod financijskih transakcija).

Znanja stečena u području optimizacije i dizajniranja eksperimenata omogućavaju zapošljavanje u raznim područjima gospodarstva, primjerice u tvrtkama koje imaju potrebu optimizirati razne proizvodne procese ili trebaju testirati gotove proizvode i prototipove.

Izborni modul Optimizacija svojim sadržajem pridonosi razvoju kompetencija studenata u području primjene diskretne matematike. U razvoju ovog izbornog modula korištena su saznanja što ih o problemima optimizacije u poslovanju privrednih subjekata (npr. optimizacija poslovnih/proizvodnih procesa) imaju partneri iz gospodarstva s kojima surađuje Odjel za matematiku.

Division of Discrete Mathematics

DISCRETE MATHEMATICS AND ITS APPLICATIONS

In addition to the existing study programmes, conducted since its establishment, the Department of Mathematics has developed the graduate program Discrete Mathematics and Its Applications which is carried out since the academic year 2011/2012. It is the only study programme in the field of discrete mathematics in Croatia.

Discrete mathematics is the branch of mathematics dealing with discrete mathematical structures, i.e. mathematical structure over countable sets (finite sets or sets with the same cardinality as the set of natural numbers).

Topics in discrete mathematics, among others, include combinatorics, graph theory, coding theory and cryptography. Concepts from discrete mathematics are especially useful in studying and describing objects and problems in branches of computer science, such as computer algorithms, programming languages, automated theorem proving, and software development.

Conversely, computer implementations are significant in applying ideas from discrete mathematics to real-world problems, such as in optimization.

The knowledge gained in this study programme is highly applicable in the economy; graph theory has broad application, from telecommunications to the design of road networks, coding theory and cryptography are used daily in communication. Since there will be more and more jobs related with ICT technologies and data protection, the need for this profile of personnel will increase. The optimization is very purposeful in various business processes, while the design and analysis of experiments are necessary in carrying out of any experiment, from the production of new drugs to machine testing. Also, the knowledge in the field of experimental design is very applicable in examining the characteristics of the finished products and we expect the labour market to demonstrate the need for our graduates.

The completion of this kind of study provides a good basis for the possible development of a scientific career in the field of science and engineering, but it also provides potential employment in various areas, at the positions which require algorithmic way of thinking and the ability to analyse data.

DISCRETE MATHEMATICS AND ITS APPLICATIONS

Title of study programme	**Discrete mathematics and its applications**
study programme coordinator	**University of Rijeka**
Study programme implementor	**Department of Mathematics – University of Rijeka**
Type of study programme	**Full-time study**
Level of study programme	**Graduate**
Academic/professional degree awarded upon completion of study	**Master of Science in Mathematics**
Duration of study programme	**2 years (4 semesters)**
Language	**English**
ECTS credits – minimal number of credits required for completion of study programme	**120 ECTS**

Required educational level for enrollment: / Bachelor's degree in mathematics or equivalent

More information:

http://www.math.uniri.hr/en/

DISCRETE MATHEMATICS AND ITS APPLICATIONS

CURRICULUM

L - lectures, E - exercises, S - seminars (contact hours)
ECTS - European Credit Transfer and Accumulation System points
STATUS: C - compulsory for the module Optimization

Year of study: 1

Semester 1

	L	E	S	ECTS	STATUS
Vector spaces 1	30	30	0	6	C

After completing this course, the students will be able to:
- describe basic examples of vector spaces and linear operators,
- solve problems related to the calculation of the rank,
- solve problems related to adjoint spaces,
- construct Jordan basis,
- apply and explain the procedure of reduction of an operator on finite dimensional vector spaces in particular problems,
- describe bacis examples of unitary spaces,
- classify main properties of bilinear forms,
- classify main properties and examples of normal operators.

Measure and Integral	30	30	0	6	C

After completing this course, the students will be able to:
- explain and use the properties of a measure and integral,
- analyse examples of a measure with a special emphasis on the Lebesgue measure,
- use and explain the convergence theorems in problem solving,
- use and explain the Fubini's theorem in problem solving,
- analyse the notions of absolute continuity and singularity of a measure and the relations between them,
- analyse the connections and differences between Riemann and Lebesgue integral.

Algebra 1	30	30	0	6	C

After completing this course, the students will be able to:
- define and analyse properties of free groups, apply an adequate method while solving problems,
- differentiate and analyse different categories, apply an adequate method while solving problems,
- define and analyse properties of modules, apply an adequate method while solving problems,
- define solvable groups and characterize them using different methods, apply an adequate method while solving problems,
- define nilpotent groups and characterize them using different methods, apply an adequate method while solving problems.

Linear programmning	30	30	0	6	C

After completing this course, the students will be able to:
- classify basic convex sets of points in n-dimensional Euclidean space and proper analytical methods of
- solving linear programming problems,
- apply properties of a linear (affine) function to a linear programming problem,
- define the goal function in simple linear programming problems,
- apply and explain various algorithms for finding extreme values of a linear function on a convex set,
- solve the dual problem of linear programming,
- apply and explain the Simplex algorithm,
- analyse the concept of matrix games,
- solve problems of integer programming,
- analyse the basics of convex programming.

Graph theory	30	15	15	6	C

After completing this course, the students will be able to:
- differentiate the basic properties of graphs and use them in solving problems,
- analyse problems of graph connectivity and related properties,
- analyse Eulerian and Hamiltonian graphs and apply the definitions and properties in solving problems,
- solve problems related to a matching in graphs.

Semester 2

	L	E	S	ECTS	STATUS
Statistics	30	30	0	6	C

After completing this course, the students will be able to:
- present statistical data in tabular and graphical form,
- explain the classification of statistical variables,
- analyse continuous random variables and vectors that are used in statistics,
- use estimators and their properties within the specific statistical models,
- using a computer, construct confidence intervals and conduct a procedure of testing statistical hypotheses,
- using a computer, apply methods of statistical data analysis.

Algebra 2	30	30	0	6	C

After completing this course, the students will be able to:
- define, give examples and recognise basic algebraic structures with two operations,
- explain the concept of ring, ideal and ring homomorphism,
- explain the basic theorems of polynomial theory,
- explain and apply various types of field extensions,
- successfully solve problems of determining Galois group,
- explain the basics of Galois theory.

Probability theory	30	30	0	6	C

After completing this course, the students will be able to:
- apply random variables and their properties in solving problems,
- explain the classification of random variables,
- apply limit theorems for mathematical expectation,
- apply basic probability inequalities,
- explain basic types of convergence of random variables and their relations,
- explain weak and strong laws of large numbers, and convergence of series of random variables,
- apply properties of characteristic functions in solving problems,
- explain inversion and continuity theorems for characteristic functions,
- explain weak convergence of sequence of distribution functions,
- apply the central limit theorem.

Artificial intelligence	30	30	0	6	C

After completing this course, the students will be able to:
- analyse different perspectives on what are the problems of artificial intelligence,
- explain the basic knowledge representation, problem solving, and learning methods of artificial Intelligence,
- assess the applicability, strengths, and weaknesses of the basic knowledge representation, problem solving, and learning methods in solving particular problems,
- develop intelligent systems through examples of concrete computational problems,
- design basic problem solving methods based on artificial intelligence - based search, reasoning, planning, and learning algorithms,

describe logic programming language associated with artificial intelligence.

Coding theory and cryptography	30	15	15	6	C

After completing this course, the students will be able to:
- differentiate and analyse cryptography systems and apply an adequate procedure in problem solving,
- analyse and differentiate type of codes and apply an adequate procedure in problem solving,
- differentiate ways of detecting errors in data transfer with a particular coding method and analyse the
- conditions for correcting these errors.

Year of study: 2.

Semester 3

	L	E	S	ECTS	STATUS
Permutation groups	30	15	15	6	C

After completing this course, the students will be able to:

- differentiate and analyse various actions of a group on a set, explain and apply adequate methods while solving problems,
- differentiate and analyse various examples of permutation groups, explain and apply adequate procedures while solving problems,
- construct various finite structures from permutation groups and analyse their properties,
- explain and apply O'Nan-Scott theorem and its consequences,
- describe the classification of finite simple groups.

Number theory	30	30	0	6	C

- After completing this course, the students will be able to:
- analyse basic properties of integers and apply those properties to simple problems in the number
- theory related to divisibility and divisibility algorithms,
- calculate using modular arithmetics, solve congruency equations and systems of congruencies,
- explain and apply the quadratic law of reciprocity and formulas for calculating the Legendre symbol,
- to solve quadratic congruencies,
- describe a display of integers by using quadratic forms in simple cases, compare and classify different quadratic forms,
- show and analyse basic multiplicative functions and their properties, check and show connections between them,
- define basic types of Diophantine equations and describe the methods of solving them,
- define elliptic curves, analyse their basic properties and describe important open problems,
- explain and apply the methods in the number theory in analysis of the public-key cryptosystem,
- describe and analyse algebraic and analytical methods in the number theory and apply them to important problems in the number theory.

Introduction to design theory	30	15	15	6	C

After completing this course, the students will be able to:
- define the basic concepts of the design theory,
- explain and apply the basic theorems of the design theory,
- construct examples of block designes and related combinatorial structures,
- apply the design theory in the elementary problems of the coding theory, threshold schemes, visual
- cryptography and group testing.

Design and analysis of experiments	30	15	15	6	C

After completing this course, the students will be able to:
- describe and apply the basic principles and methods for designing and analysing experiments to particular examples,
- analyse the model for designs with one source of variation,
- analyse and apply the methods of multiple comparisons,
- analyse models for two treatment factors,
- use the appropriate software package for solving problems,
- analyse basic notions in statistical design theory,
- apply the basic notions in statistical design theory to particular examples.

Nonlinear optimization	30	30	0	6	CM

After completing this course, the students will be able to:
- list different methods of nonlinear optimization,
  formulate problems in nonlinear optimization and appreciate their assumptions and limitations,
- choose an appropriate method for solving nonlinear optimization problem using modern optimization methods and software.

Semester 4

	L	E	S	ECTS	STATUS
Seminar of M.Sc. thesis	0	0	30	4	C

After completing this course, the students will be able to:
- present mathematical concepts using teaching aids and facilities,
- express correctly and fluently in speaking communication in the language of teaching,
- use different communication types and forms,
- use relevant and recent professional literature independently and critically.

Combinatorial optimization	30	30	0	6	CM

After completing this course, the students will be able to:
- list different mehods of combinatorial optimization,
- differ optimal and heuristic methods of combinatorial optimization (i.e. optimal and near-optimal solutions),
- formulate problems in combinatorial optimization and appreciate their assumptions and limitations,
- choose appropriate method for solving combinatorial optimization problem using modern optimization methods and software.

Machine learning	30	30	0	6	CM

After completing this course, the students will be able to:
- describe machine learning techniques and computing environment that are suitable for the applications,
- analyse various types of learning algorithms,
- develop machine learning techniques and associated computing techniques and technologies for various applications,
- identify current real world problems that can benefit from emerging machine learning techniques,
- design machine learning and associated algorithms that can address real problem.

Optimization techniques for data mining	30	15	15	5	CM

After completing this course, the students will be able to:
- describe data mining techniques,
- analyse different types of algorithms in data mining,
- use some techniques of data mining in practice,
- design algorithms in data mining that can address real problem.

Optimization methods in finance	30	15	15	5	CM

After completing this course, the students will be able to:
- define basic terms related to financial mathematics,
- list different optimization mehods in finance,
- formulate problems in financial mathematics and appreciate their assumptions and limitations,
- solve practical problems arising in finance using modern optimization methods and software.

Graduation				4	C

ALUMNI

In 2013, Borka Frančišković graduated from the Graduate University Study of Discrete Mathematics and Its Applications. She works at the Zagrebačka Bank in Zagreb as a quantitative credit risk analyst, which includes work on the development, improvement and implementation of systems related to credit risk parameters. She is also responsible for calculation of internal capital and application of stress tests for credit risk.

About the completed study says: "... Graduate University Study Discrete Mathematics and Its Applications, apart from qualifying for mathematics and other science studies, also provides the necessary knowledge to carry out optimization and application of statistics, applicable in various branches of economy such as banking. Courses of Discrete mathematics, on which basis development of models for measuring credit risk parameters is built on, encourage the development of logical thinking and strategies for dealing with wide-ranging problems. Also, we have the benefit of seminars through the whole course of study, which have enabled us to explain a subject to the interlocutor in a clear and easy way. "

In 2014, Hana Rizvić graduated from the Graduate University Study of Discrete Mathematics and Its Applications. After completing her studies, she was employed at iOLAP Coadria in Rijeka. She first worked as a Business Intelligence Developer, and later on continued her career as a Data Scientist, at the same company.

*About the completed study says: " ... almost all of mathematical tasks at my work are solved through R and Python programs. My math studies are used mostly as a theoretical basis for the work I am doing today .... I use the knowledge acquired at courses such as Database and Probability and Statistics, the programming I have taught through courses such as Introduction to Design Theory and Permutation Group ... to re-enroll my studies today, I would study the same***."**

Tin Zrinski has obtained his master's degree in Discrete Mathematics and Its Applications in 2016. After graduation, he was hired as a research and teaching assistant at the Department of Mathematics, University of Rijeka. He enrolled in the joint PhD program in Mathematics, held by J.J.Strossmayer University of Osijek, Univerity of Rijeka, Univerity of Split and University of Zagreb. He is a member of the Finite Mathematics Seminar at the Department of Mathematics, University of Rijeka and his research interest is in the field of discrete and combinatorial mathematics.

About the completed study says: „The graduate study programme Discrete Mathematics and Its Applications at the Department of Mathematics trains students for different types of jobs in the field of mathematics, but also in the field of informatics. For me, this programme opened up the possibility of further advancement in the field of mathematics, through the doctoral study I enrolled in as a part of the research and teaching assistant position at the Department of Mathematics. Also, these studies have qualified me to engage in research work in discrete mathematics, a fulfilling part of my job in which I apply the knowledge acquired throughout the study, in particular the programming skills and the use of computer for the purpose of solving mathematical problems. Another part of my job is passing on my own knowledge to new generations of students, where the experience of holding various seminars during the study has proved helpful. I also participate in workshops whose purpose is popularization of mathematics for students in elementary and high schools where we try to bring closer to the students some mathematical topics which are not part of their school curriculum. I recommend to enroll in this study because it opens up the possibility of quick employment in creative and dynamic jobs of different profiles."

PREDDIPLOMSKI SVEUČILIŠNI STUDIJ MATEMATIKE

3 godine (6 semestara)

180 ECTS bodova

Sveučilišni prvostupnik (baccalaureus) matematike

Sveučilišna prvostupnica (baccalaurea) matematike

DIPLOMSKI SVEUČILIŠNI STUDIJ MATEMATIKE

(smjer: nastavnički)

2 godine (4 semestra)

120 ECTS bodova

Magistar edukacije matematike

Magistra edukacije matematike

DIPLOMSKI SVEUČILIŠNI STUDIJ MATEMATIKA I INFORMATIKA

(smjer: nastavnički)

2 godine (4 semestra)

120 ECTS bodova

Magistar edukacije matematike i informatike

Magistra edukacije matematike i informatike

DIPLOMSKI SVEUČILIŠNI STUDIIJ DISKRETNA MATEMATIKA I PRIMJENA

2 godine (4 semestra)

120 ECTS bodova

Magistar matematike

Magistra matematike

POSLIJEDIPLOMSKI SVEUČILIŠNI STUDIJ MATEMATIKA

(Sveučilište Josipa Jurja Strossmayera u Osijeku, Sveučilište u Rijeci, Sveučilište u Splitu, Sveučilište u Zagrebu)

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Doktor / Doktorica znanosti iz polja matematike

Struktura sveučilišnih studijskih programa koje izvodi Odjel za matematiku (po ciklusima)

UNDERGRADUATE COURSE IN MATHEMATICS

3 years (6 semesters)

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GRADUATE COURSE IN MATHEMATICS

(teacher training)
2 years (4 semesters)

120 ECTS credits

Master of Science in Mathematics Education

GRADUATE COURSE IN MATHEMATICS AND COMPUTER SCIENCES

(teacher training)

2 years (4 semesters)

120 ECTS credits

Master of Science in Mathematics and Computer Science Education

GRADUATE COURSE IN DISCRETE MATHEMATICS AND ITS APPLICATIONS

2 years (4 semesters)

120 ECTS credits

Master of Science in Mathematics

DOCTORAL STUDY OF MATHEMATICS

(J.J.Strossmayer University of Osijek, Univerity of Rijeka, Univerity of Split and University of Zagreb)

3 year (6 semesters)

180 ECTS credits

Doctor of Mathematics (PhD)

The structure of the study programmes carried out by the Department of Mathematics

Sveučilište u Rijeci
University of Rijeka

Trg braće Mažuranića 10, 51 000 Rijeka, Croatia
Tel: +385 (0)51 406500
W: www.uniri.hr
E: esf_ivo@uniri.hr

UNIVERSITY OF RIJEKA
DEPARTMENT OF MATHEMATICS

Department of Mathematics - University of Rijeka
Odjel za matematiku Sveučilišta u Rijeci

Radmile Matejčić 2, 51000 Rijeka, Croatia
Tel: +385 (0)51 584 650
Fax: +385 (0)51 584 699
E: math@math.uniri.hr
W: www.math.uniri.hr/en

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