KOOPMAN OPERATOR THEORY: FUNDAMENTALS, APPROXIMATIONS AND APPLICATIONS 2

10-12/9/2025

CRES CROATIA

DATE&VENUE

10–12/9/2025
MOISE PALACE
CRES, CROATIA
Zagrad 6, 51557 Cres

Working language: English

In recent years, the development of Koopman operator theory has led to applications across various research fields, including robotics, biology, climate science, and artificial intelligence. Significant theoretical and numerical advancements have accelerated progress in these domains. Furthermore, the expansion of Koopman operator methods into new application areas has introduced novel theoretical and computational challenges.

The goal of this workshop is to bring together researchers from diverse areas of Koopman-operator-driven research. Topics will include recent advances in robotics and biological applications, innovations in climate science leveraging Koopman operator methods, control-oriented applications, and emerging applications in artificial intelligence. Additionally, the workshop will explore how insights from these applications contribute to theoretical developments in the field.

The expected outcome of the workshop is a structured roadmap that outlines key research challenges, recent advancements, and future directions in Koopman Operator Theory, guiding the community in its ongoing and future efforts.

ORGANIZERS

10/9/25
MOISE PALACE, CRES

TIME	EVENT	DESCRIPTION
8:30–9:00	REGISTRATION	
9:00–9:40	WELCOME	Igor Mezić Antonio Navarra Ervin Kamenar Andrea Mešanović Marin Gregorović, Mayor of Cres Lado Kranjčević
9:40–10:55	SESSION 1 : KOOPMAN THEORY FOR DYNAMICAL SYSTEMS AND CONTROL I	Igor Mezić Dimitrios Giannakis Yoshihiko Susuki
10:55–11:10	DISCUSSION	
11:10–11:40	COFFEE BREAK	
11:40–12:55	SESSION 2 : KOOPMAN THEORY FOR CONTROL	Mircea Lazar Hiroya Nakao Karl Worthmann
12:55–13:10	DISCUSSION	
13:10–14:40	LUNCH BREAK	Location: Nono Frane
14:40–15:55	SESSION 3 : KOOPMAN THEORY NUMERICS I	Caroline Wormell Zlatko Drmač Ela Đimoti
15:55–16:10	DISCUSSION	
16:10–16:40	COFFEE BREAK	
16:40–17:55	SESSION 4 : APPROXIMATIONS AND ESTIMATIONS OF THE KOOPMAN OPERATOR	Nelida Črnjarić Alexandre Mauroy
17:30–17:45	DISCUSSION	
17:45–18:00	GENERAL DISCUSSION & END OF WORKSHOP	
19:30	NETWORKING DINNER	Location: Konoba Kumpanija

WEDNESDAY

11/9/25
MOISE
PALACE,
CRES

TIME	EVENT	DESCRIPTION
8:30–8:45	REGISTRATION	
8:45–9:15	WELCOME	Senka Maćešić
9:15–10:30	SESSION 1 : KOOPMAN THEORY APPLICATIONS: CLIMATE & ENVIRONEMNETAL SCIENCES	Antonio Navarra Giulia Libero Paula Sanchez Valerio Lucarini
10:30–10:45	DISCUSSION	
10:45–11:15	COFFEE BREAK	
11:15–12:55	SESSION 2 : KOOPMAN THEORY APPLICATIONS: ROBOTICS	Joel Burdick Ervin Kamenar Tomislav Bazina
12:55–13:10	DISCUSSION	
13:10–14:40	LUNCH BREAK	Location: Nono Frane
15:30–16:30	BOAT EXCURSION	Boarding point: here
16:30–17:00	YOUNG RESEARCHERS DISCUSSION ON THE BOAT Moderator: Marko Kozlov	
17:30–19:00	DINNER	Location: On a boat

THURSDAY

12/9/25
MOISE PALACE, CRES

TIME	EVENT	DESCRIPTION
9:00–9:15	REGISTRATION	
9:15–10:55	SESSION 1 : KOOPMAN THEORY FOR DYNAMICAL SYSTEMS AND CONTROL II	Harry Asada Jeff Moehlis Petar Bevanda Matthew Kvalheim
10:55–11:10	DISCUSSION	
11:10–11:40	COFFEE BREAK	
11:40–12:30	SESSION 2 : KOOPMAN THEORY FOR MACHINE LEARNING AND AI	William Redman Alan Avilla
12:30–12:45	DISCUSSION	
13:10–14:40	LUNCH BREAK	Location: Nono Frane
16:30–17:00	SESSION 3 : KOOPMAN THEORY FOR BIOLOGICAL SYSTEMS AND HEALTHCARE	Enoch Yeung Xin Yee David Liovic
15:55–16:10	DISCUSSION	
16:10–16:40	COFFEE BREAK	Location: On a boat
16:40–17:55	SESSION 4 : KOOPMAN THEORY NUMERICS II	Stefan Klus Andrew Horning Ioannis Kevrekidis
17:55–18:10	DISCUSSION	
18:10–18:30	GENERAL DISCUSSION & END OF WORKSHOP	
19:30	NETWORKING DINNER	Location: Grill&BBQ House Porta Bragadina

FRIDAY

SPEAKER	TALK TITLE	ABSTRACT
IGOR MEZIĆ	KOOPMAN OPERATOR THEORY IN DYNAMICAL SYSTEMS, NEURAL NETWORKS AND CONTROL	I will provide an overview of the current set of efforts and open problems in Koopman Operator Theory. Its use in machine learning is becoming widespread and I will discuss the various frameworks as well as applications of the Koopman Operator in neural network training. I will present the theory of the "static Koopman operator" and describe its applications, promising to extend the reach of the operator theoretic approach to machine learning of input-output maps. Finally, I will discuss some recent results in the fast expanding field of Koopman Operator-based Control Theory. Acknowledgement: Support from ARO, AFOSR, DARPA, NSF and ONR is gratefully acknowledged.
ANTONIO NAVARRA	THE RISE OF KOOPMAN THEORY IN CLIMATE SCIENCE	Koopman theory presents a potential new interpretation for the dynamics of complex systems, such as the Earth. In this discussion, we explore the potential of Koopman operator theory and its adjoint to provide a fresh and powerful approach to investigating the relationship between climate variability and tropical and global sea surface temperatures (SSTs). We demonstrate that the Koopman modes yield a decomposition of the data sets that can be used to categorize the variability. Notably, the most relevant modes emerge naturally and can be easily identified. The segmentation of the space of eigenfunctions into stationary and non-stationary modes offers an intriguing opportunity to investigate the scaffolding of the phase space around which the dynamics unfolds.
ZLATKO DRMAČ	ADVANCES IN DMD – USING THE RESIDUALS AND THE KOOPMAN-SCHUR DECOMPOSITION	We present two themes from recent development of the Dynamic Mode Decomposition (DMD). First, the DMD is presented as a data driven Rayleigh-Ritz extraction of spectral information. This (unlike the mere regression aspect) allows for a better connection with the Koopman operator, and provides better understanding of the dynamics under study. Computable residuals can be used to select physically meaningful eigenvalues and modes, and to guide sparse representation of the snapshot in the KMD (Koopman Mode Decomposition). We believe that this is the proper approach to the DMD as a numerical toolbox for Koopman operator based data driven analysis of nonlinear dynamics. In some cases, numerically computed compression of the Koopman operator exhibits high non-normality, and, as a result, the eigenvectors are highly ill-conditioned and the KMD becomes numerically unstable. We address this problem and introduce a new theoretical and computational framework for data driven Koopman mode analysis of nonlinear dynamics. The problem of ill-conditioned eigenvectors is solved using a Koopman-Schur decomposition that is entirely based on unitary transformations. The analysis in terms of the eigenvectors as modes of a Koopman operator is replaced with a modal decomposition in terms of a flag of invariant subspaces that correspond to selected eigenvalues. The new approach has the same functionalities in terms of representation of data snapshots and forecasting.

ALEXANDRE MAUROY

IDENTIFICATION, SPECTRAL ANALYSIS, AND STATE ESTIMATION IN SPACES OF ANALYTIC FUNCTIONS

We will present a novel EDMD-type method that identifies a finite-dimensional representation of the Koopman operator defined on a reproducing kernel Hilbert space of analytic functions. Our approach relies on an orthogonal projection onto a polynomial subspace, which is equivalent to data-driven Taylor approximation. In the case of dynamics with a hyperbolic equilibrium, the proposed method demonstrates excellent performance to capture the Taylor expansion of Koopman eigenfunctions and the Koopman spectrum, including the eigenvalues of the linearized system at the equilibrium. Moreover, the technique does not suffer from spectral pollution, reaching arbitrary accuracy on the spectrum with a fixed finite dimension of the approximation.

If time permits, we will show how a Luenberger observer can be built to estimate the (infinite-dimensional) state of a dual Koopman system, and equivalently the (finite-dimensional) state of the nonlinear dynamics. This latter result supports and extends numerical Koopman operator-based estimation techniques known from the literature.

YOSHI SUSUKI

KOOPMAN OPERATORS FOR SINGULAR SYSTEMS: CONSTRUCTION, SPECTRAL PROPERTIES, AND DATA-DRIVEN METHODS

Singular forms of dynamical systems, including singularly perturbed ODEs exhibiting multiple timescale dynamics and differential-algebraic equations exhibiting dynamics constrained on manifolds, have been studied in nonlinear theory and its applications. I will discuss our recent work on the Koopman operator framework for the singular systems. Our research aims to provide an alternative viewpoint to analyzing and synthesizing such singular systems, different from the traditional differential-geometric viewpoint. The work discussed here will include the construction of Koopman operators for a suitable space of observables, their spectral properties connected to regular and irregular dynamics arising in the systems, and numerical methods for estimating the spectral properties directly from time-series data. If time is allowed, I will introduce their application to engineered systems, such as the electric power grid. The contents are joint work with Mr. Natsuki Katayama and Mr. Taishi Yoshimura (Kyoto University, Japan).

ERVIN KAMENAR

KOOPMAN-BASED PREDICTION OF HAND GRIP FORCE USING SEMG

Conditions like stroke or multiple sclerosis impair hand function, greatly limiting an affected person's daily activities. While traditional rehabilitation methods, which often involve physiotherapists, lack the necessary intensity, robotic rehabilitation significantly enhances outcomes in restoring hand function. Innovative methods utilizing surface electromyography (sEMG) further adapt the device's force output to the user's needs, thereby enhancing rehabilitation outcomes. Leveraging flexible signal-processing steps and a novel data-driven approach based on Koopman operator theory, coupled with problem-specific data lifting techniques, this study aims to achieve accurate force estimations and predictions during medium wrap grasps using sEMG measurements. Thirteen healthy volunteers performed grip tasks at five force levels while sEMG was recorded from two forearm positions, and ground-truth force was measured using a dynamometer. Optimal signal processing parameters were identified through multi-step sensitivity analysis. The Koopman-based methodology for estimating and short-term predicting grip force from processed sEMG signals proved robust with respect to precise electrode positioning. The algorithm executes exceptionally fast, processing, estimating, and predicting a 0.5-second sEMG signal batch in a 30-ms time frame, facilitating real-time implementation.

HARRY ASADA

KOOPMAN GLOBAL LINEARIZATION OF CONTACT-RICH DYNAMICS ENABLES ROBOTS TO DISCOVER ELABORATE CONTROL STRATEGIES FOR LOCOMOTION AND MANIPULATION

Controlling robots that dynamically engage in contact is a pressing challenge in robotics. Whether a legged robot making-and-breaking contact with the ground during locomotion, or a manipulator grasping objects, contact is everywhere and the key to control synthesis. Unfortunately, the hybrid nature of contact dynamics with governing equations segmented into multiple regions makes control difficult. Model Predictive Control (MPC), for example, faces non-convex optimization when contact is involved. In this talk, a Koopman-based alternative approach to the modeling, prediction, and control of complex robot systems that make-and-break contact will be presented. It will be shown that a globally linear, unified model can subsume a group of segmented dynamics and provides MPC-amenable representations of otherwise highly complex, switched nonlinear systems. To make this Koopman-based modeling possible, two fundamental challenges must be overcome. One is that, strictly speaking, the original Koopman operator theory is applicable only to autonomous systems with no control input. The second fundamental challenge is that the existence of a Koopman operator cannot be guaranteed for dynamic systems with discontinuities. For the first challenge, an emerging method, termed Control-Coherent Koopman (CCK) modeling, will be introduced. CCK utilizes an inherent property of actuator dynamics, where input terms appear linearly in the governing nonlinear dynamics. The resultant CCK model does not require approximation of the control matrix in the globally linear state equation in the lifted space. For the second challenge, conventional discontinuous contact modeling will be replaced by causal physical modeling, which is continuous. It will be shown that viscoelastic models of contact are not only natural, realistic, and accurate but can also satisfy one of the key conditions on CCK formulation. The proposed CCK-MPC control system can predict trajectories that include a series of contact changes over a time horizon and find an optimal control sequence traversing multiple regions of segmented dynamics in the original state space. The method is applied to both locomotion and manipulation: two major fields of robotics. Unlike its traditional counterparts, the CCK-MPC does not need a pre-determined sequence of contact changes but can find an optimal one in real time. The method is verified through both simulation and experiment. The real-time computation in manipulation experiments requires only less than 8 milli seconds on a standard PC. The Koopman operator theory allows for the unified, globally-linear modeling that enables the robot to discover elaborate MPC control strategies at an over 100Hz sampling rate. The method is applicable to broad fields beyond robotics where governing dynamics are segmented.

IOANNIS KEVREKIDIS

KOOPMAN EIGENFUNCTIONS AND INFINITIES

If the flow map of a dynamical system is invertible, the eigenfunctions of the Koopman operator of the system form a group. Here, we exploit this property to speed up the numerical computation of the eigenspaces of the operator. Given a small set of (so-called ``principal'') eigenfunctions that are approximated conventionally, we construct polynomials of these eigenfunctions to obtain a much larger set. Often, eigenfunctions exhibit localized singularities (e.g., in a simple one-dimensional saddle-node problem); we discuss eigenfunctions continuation across such singularities.

TITLES & ABSTRACTS

JOEL BURDICK

REAL-TIME KOOPMAN-BASED DYNAMICAL PREDICTION (FROM NOISY SIGNALS) FOR SAFETY-CRITICAL CONTROL OF AUTONOMOUS SYSTEMS.

Many autonomous systems (e.g, driverless cars and drones) must make decisions based on future predictions of the actions of other nearby agents, whose dynamics and intentions are unknown. E.g., autonomous cars must predict the motions of surrounding vehicles, pedestrians and bicycles. Autonomous racing drones must avoid crashing into other drones on the race course. Unfortunately, only partial and noisy data on the motions of these potential hazards are available. This talk will introduce a novel method to approximate a predictive Koopman operator for each potential hazard from noisy data, quantify the uncertainty of the future predictions, and use the quantified predictions to provide probabilistic collision avoidance guarantees within a real-time model predictive control framework.

NELIDA ČRNJARIĆ

KOOPMANIZING MICROPOLAR FLUID DYNAMICS: A DATA-DRIVEN EXPLORATION

This work investigates the potential of Koopman operator-based methods for analyzing, reducing, and predicting the behavior of micropolar fluid flows. The equations for compressible micropolar fluids extend the classical Navier—Stokes framework by incorporating, in addition to standard flow phenomena, microrotational dynamics and internal moments. We apply data-driven techniques, including DMD and extended DMD, to learn approximate Koopman operators from numerical simulations. The study focuses on reduced-order modeling, reconstruction of full state dynamics from limited observables, and long-term prediction of nonlinear flow evolution.

We evaluate the performance of these methods on both classical compressible and micropolar fluid model. Our results highlight the additional spectral richness introduced by micropolar terms. This analysis provides insights into the applicability and limitations of Koopman theory for modeling complex continuum systems.

DIMITRIOS GIANNAKIS

QUANTUM MECHANICAL CLOSURE OF PARTIAL DIFFERENTIAL EQUATIONS WITH SYMMETRIES

We present a framework for closure of spatiotemporal dynamics governed by partial differential equations based on quantum mechanics. This approach builds quantum mechanical systems that are embedded within the discretization mesh of the coarse model, and act as data-driven surrogate models for the unresolved degrees of freedom of the original dynamics. The coupled classical-quantum system implements a prediction-correction cycle, whereby the quantum mechanical models provide estimates of the subgrid fluxes through expectation values of quantum observables evolving under the Koopman operator, and the classical coarse model conditions the quantum mechanical states via a quantum mechanical Bayes rule. Moreover, by combining methods from operator-valued kernels and delay embedding, the quantum mechanical systems capture a compressed representation of the dynamics that is positivity-preserving and invariant under spatial symmetries of the original dynamics. We illustrate this scheme with applications to closure of the shallow-water equations and cloud-resolving atmospheric convection models.

TITLES & ABSTRACTS

VALERIO LUCARINI

INTERPRETABLE AND EQUATION-FREE RESPONSE THEORY FOR COMPLEX SYSTEMS

Response theory provides a pathway for understanding the sensitivity of a system and for predicting how its statistical properties change when a perturbation is applied. In the case of complex and multiscale systems, to achieve enhanced practical applicability, response theory should be interpretable, capable of focusing on relevant timescales, and amenable to data-driven and equation-agnostic implementations. Along these lines, in the spirit of Markov state modelling, we present linear and nonlinear response formulas for Markov chains. We obtain simple and easily implementable expressions that can be used to predict the response of observables as well as of higher-order correlations. The methodology proposed here can be implemented in a purely data-driven setting and even if the underlying evolution equations are unknown. The use of algebraic expansions inspired by Koopmanism allows to elucidate the role of different time scales and modes of variability, and to find explicit and interpretable expressions for the Green's functions at all orders. This is a major advantage of the framework proposed here. We illustrate our methodology in a very simple yet instructive metastable system. Finally, our results provide a dynamical foundation for the Prony method, which is commonly used for the statistical analysis of discrete time signals.

JEFF MOEHLIS

PHASE REDUCTION, FOR THE WIN!

In mathematical neuroscience, a common tool for understanding the dynamics and designing the control of neurons is phase reduction. This dates back to the work of Art Winfree in the 1960's, who introduced the notion of an asymptotic phase. The key insight here was that the behavior of a general limit cycle oscillator in response to perturbations could be characterized in terms of the timing of oscillations rather than in reference to the underlying state variables. In subsequent years, these ideas were formalized using the notion of isochrons, i.e., level sets of initial conditions that have the same asymptotic convergence to the limit cycle. By restricting one's attention to a small neighborhood of the limit cycle, one can obtain a 1-dimensional phase reduced equation that can accurately capture the response to weak perturbations. Such phase reductions have been used for upwards of half a century to understand complex emergent patterns in weakly perturbed neurons and other oscillators. More recent applications have highlighted the usefulness of also considering the amplitude or isostable dynamics in a phase reduced coordinate framework in order to extend applicability beyond the weakly perturbed paradigm. It is worth emphasizing that isochrons and isostables can be interpreted in terms of eigenfunctions of the Koopman operator.

In this talk, I will describe several control problems in mathematical neuroscience that have been usefully formulated in terms of phase reduction. This will include changing the phase of an oscillator using a minimum energy input, and desynchronizing populations of neural oscillators using optimal chaotic desynchronization, optimal phase resetting, and phase distribution control. It is hoped that this talk will demonstrate to the broader Koopman community the utility of phase reduction for controlling neurons and other oscillators.

HIROYA NAKAO

KOOPMAN OPERATOR APPROACH TO LIMIT CYCLES IN QUANTUM DISSIPATIVE SYSTEMS

The Koopman operator has provided new insights into dynamical reduction of limit-cycle oscillators. The asymptotic phase, originally a geometrical concept, can now be defined as the argument of the principal Koopman eigenfunction with a pure imaginary eigenvalue. Moreover, the amplitudes characterizing deviations from the limit cycle can be defined as principal eigenfunctions with eigenvalues having negative real parts. This has led to a generalization of the classical phase reduction theory into a phase-amplitude reduction theory. Recently, the synchronization of quantum dissipative limit-cycle oscillators has attracted interest. In the semiclassical regime, the oscillator dynamics can be approximated by a classical stochastic differential equation, with noise arising from quantum fluctuations. The asymptotic phase in this case can be defined as the eigenfunction of the associated stochastic Koopman operator, i.e., the backward Kolmogorov operator. In the fully quantum regime, the semiclassical approximation is no longer valid, but phase and amplitude-like quantities can still be defined via the principal eigenfunctions of the backward quantum Liouville operator. This talk will briefly outline phase-amplitude reduction of deterministic limit cycles, its extension to stochastic or semiclassical systems, and possible generalizations to fully quantum limit cycles.

KARL WORTHMANN

KERNEL APPROXIMANTS OF THE KOOPMAN OPERATOR AND THEIR USE IN PREDICTIVE CONTROL

Extended dynamic mode decomposition [1; EDMD], embedded in the Koopman framework [2], is a widely-applied technique for predictive control of dynamical control systems [3]. However, despite its popularity, the error analysis is still fragmentary. We provide an analysis of the full approximation error [4]. To this end, we show Koopman invariance of suitably-constructed reproducing kernel Hilbert spaces (RKHS) and leverage approximation-theoretic arguments for (regularised) kernel EDMD. Then, we extend the results to control-affine systems and demonstrate the applicability of the EDMD surrogate models for model predictive control [6] as well as data-driven controller design of nonlinear systems [7].

[1] M.O. Williams, I.G. Kevrekidis, C.W. Rowley: A data–driven approximation of the koopman operator: Extending dynamic mode decomposition, Journal of Nonlinear Science 25:1307-1346, 2015.
[2] I. Mezić: Spectral properties of dynamical systems, model reduction and decompositions. Nonlinear Dynamics 41:309-325, 2005.
[3] M. Korda and Igor Mezić: On convergence of extended dynamic mode decomposition to the Koopman operator, Journal of Nonlinear Science 28:687-710, 2018.
[4] F. Köhne, F.M. Philipp, M. Schaller, A. Schiela, K. Worthmann: L_∞-error bounds for approximations of the Koopman operator by kernel extended dynamic mode decomposition, SIAM Journal on Applied Dynamical Systems 24(1):501–529, 2025.
[5] I. Schimperna, K. Worthmann, M. Schaller, L. Bold, L. Magni: Data-driven Model Predictive Control: Asymptotic Stability despite Approximation Errors exemplified in the Koopman framework, arXiv:2505.05951.
[6] R. Strässer, M. Schaller, K. Worthmann, J. Berberich, F. Allgöwer; Koopman-based feedback design with stability guarantees. IEEE Transactions on Automatic Control 70(1).355-370, 2025.

STEFAN KLUS

DATA-DRIVEN SYSTEM IDENTIFICATION USING QUADRATIC EMBEDDINGS OF NONLINEAR DYNAMICS

We propose a novel data-driven method called QENDy (Quadratic Embedding of Nonlinear Dynamics) that not only allows us to learn quadratic representations of highly nonlinear dynamical systems, but also to identify the governing equations. The approach is based on an embedding of the system into a higher-dimensional feature space in which the dynamics become quadratic. Just like SINDy (Sparse Identification of Nonlinear Dynamics), our method requires trajectory data and a set of preselected basis functions, also called dictionary. We illustrate the efficacy and accuracy of QENDy with the aid of various benchmark problems and compare its performance with SINDy and a deep learning method for identifying quadratic embeddings. Furthermore, we analyze the convergence of QENDy and SINDy in the infinite data limit, highlight their similarities and main differences, and compare the quadratic embedding with linearization techniques based on the Koopman operator.

LAZAR MIRCEA

FROM PRODUCT HILBERT SPACES TO THE GENERALIZED KOOPMAN OPERATOR AND THE NONLINEAR FUNDAMENTAL LEMMA

The generalization of the Koopman operator to systems with control input and the derivation of a nonlinear fundamental lemma are two open problems that play a key role in the development of data-driven control methods for nonlinear systems. Both problems hinge on the construction of observable or basis functions and their corresponding Hilbert space that enable an infinite-dimensional, linear system representation. In this talk we present a novel solution to these problems based on orthonormal expansion in a product Hilbert space constructed as the tensor product between the Hilbert spaces of the state and input observable functions, respectively. We prove that there exists an infinite-dimensional linear operator, i.e. the generalized Koopman operator, from the constructed product Hilbert space to the Hilbert space corresponding to the lifted state propagated forward in time. We further briefly outline scalable data-driven methods for computing finite-dimensional approximations of generalized Koopman operators using several choices of observable functions (kernel functions, deep neural networks and Takens' delay embeddings). Moreover, we briefly outline the derivation of a nonlinear fundamental lemma by exploiting the bilinear structure of the infinite-dimensional generalized Koopman model.

TITLES & ABSTRACTS

CAROLINE WORMELL

PROBABILISTIC EIGENVALUE LOCATION FOR OPERATORS SAMPLED FROM DATA

In random or chaotic systems, the eigenvalues and corresponding eigenspaces of (E)DMD matrices are of prime interest, as they capture slowly decaying patterns in the dynamics. These matrices are commonly approximated from randomly or ergodically sampled data, but there is massive variation in the stability of these eigenvalues to sampling error. This makes it difficult to objectively assess to what extent eigenvalues and eigenspaces correspond to real patterns in the data, whether they are mixed with other eigenspaces, or simply products of sampling errors.

In this talk I will introduce a data-driven indicator that allows us to rigorously locate and separate elements of the Koopman matrix's spectrum. Like a probabilistic pseudospectrum, this indicator allows us to identify which points in the spectrum are likely to be close to their "true" value and which ones are probably sampling artefacts. Importantly, it is constructed to be independent of function space norm, and for this reason is demonstrably quite efficient. It can be applied to study any operator satisfying a least-squares problem.

ENOCH YEUNG

HETEROGENOUS MIXTURES OF DICTIONARY FUNCTIONS FOR FINITE, APPROXIMATE KOOPMAN CLOSURE : WHY DEEP KOOPMAN OPERATORS WORK AND IMPLICIT TRADEOFFS BETWEEN IDENTIFICATION AND CONTROL

Koopman operators model nonlinear dynamics as a linear dynamic system acting on a nonlinear function as the state. This nonstandard state is often called a Koopman observable and is usually approximated numerically by a superposition of functions drawn from a dictionary. In our previous work, we had proposed to use deep learning combined with EDMD to learn novel dictionary functions in an algorithm called deep dynamic mode decomposition (deepDMD). The learned representation both (1) accurately models and (2) scales well with the dimension of the original nonlinear system. In this talk we analyze the learned dictionaries from deepDMD and explore the theoretical basis for their strong performance. We discover a novel class of dictionary functions to approximate Koopman observables. Error analysis of these dictionary functions show they satisfy a property of subspace approximation, which we define as uniform finite approximate closure. We discover that structured mixing of heterogeneous dictionary functions drawn from different classes of nonlinear functions achieve the same accuracy and dimensional scaling as deepDMD. This mixed dictionary does so with an order of magnitude reduction in parameters, while maintaining geometric interpretability. Our results provide a hypothesis to explain the success of deep neural networks in learning numerical approximations to Koopman operators. The subspace approximation equations likewise provide a new avenue for interpreting the relationship between identification and control for data-driven approximations of Koopman operators. We demonstrate examples of state-feedback controllers to illustrate that, when control is the objective, the Koopman model becomes secondary and subject to the control design process.

ANDREW HORNING

RIGGED DMD: DATA-DRIVEN GENERALIZED EIGENFUNCTION DECOMPOSITIONS FOR KOOPMAN OPERATORS

Koopman operator theory provides a powerful framework for data-driven analysis of nonlinear dynamical systems. At its heart, the Koopman operator governs the evolution of observables on the state-space and allows one to construct reduced-order models, extract coherent features of the dynamics, and more. However, even state-of-the-art Koopman approaches can struggle to capture important features of dynamical systems with continuous spectrum. In this talk, we show how to rigorously compute coherent features of measure-preserving dynamical systems with continuous spectrum. Our algorithm, Rigged DMD, uses a measure-preserving Dynamic Mode Decomposition to construct carefully regularized wave-packet approximations to the generalized eigenfunctions of unitary Koopman operators in rigged Hilbert spaces. We discuss the basic convergence properties of the algorithm and illustrate with a number of examples, including the nonlinear pendulum, the Lorenz system, and a high-Reynolds number lid-driven flow in a two-dimensional cavity.

TITLES & ABSTRACTS

MATTHEW KVALHEIM

WHEN DO KOOPMAN EMBEDDINGS EXIST?

Given scalar observations of an unknown nonlinear dynamical system, Extended Dynamic Mode Decomposition (eDMD) seeks to model the time-evolution of these observables by a linear dynamical system. To avoid information loss and numerical challenges, one often wants the collection of observables to separate points and be continuous (or smoother). Stacking such observables into a vector-valued function yields a globally linearizing "Koopman embedding" of the nonlinear system, which, conceptually, embeds the nonlinear system as an invariant subset of a linear system of greater or equal dimension. However, Koopman embeddings do not generally exist, posing a fundamental challenge for eDMD and related algorithms. In this talk, I will present necessary and sufficient conditions for the existence of Koopman embeddings for a broad class of nonlinear systems. Time permitting, I will also present a counterexample to the oft-repeated claim that nonlinear systems with multiple isolated equilibria do not admit a smooth (or even continuous) Koopman embedding. Aspects of this talk are based on joint work with Philip Arathoon and with Eduardo D. Sontag.

GIULIA LIBERO

RECONSTRUCTING SPATIOTEMPORAL DYNAMICS OF TOTAL WATER STORAGE ANOMALIES USING DMD

Recent advancements in satellite technology have produced environmental data with improved spatial coverage and temporal resolution. This progress requires the development of methods to extract actionable information from large datasets. This study investigates the application of dynamic mode decomposition (DMD) for identifying the dynamics of spatially correlated structures in global-scale datasets, specifically total water storage anomalies (TWSA) observed by the Gravity Recovery and Climate Experiment (GRACE) satellite missions. The results indicate that DMD facilitates data compression and enables extrapolation from a reduced set of dominant spatiotemporal structures. The method maintains the accuracy of global system dynamics predictions when reconstructing local time series. Additionally, DMD distinguishes between modes associated with periodic dynamics, such as precipitation-driven seasonal cycles and multi-year variations, and those reflecting trend effects that indicate a progressive decline in TWSA. The analysis focuses on the latter to examine patterns related to extreme TWSA values and their intensification over time. These findings highlight the potential of DMD for analyzing remote-sensing data in hydrologic research.

XIN C. YEE

KOOPMAN LINEAR QUADRATIC REGULATOR CONTROL OF MICROBUBBLE OSCILLATIONS FOR BIOMEDICAL APPLICATIONS

Microbubbles are used in biomedicine for both diagnostic and therapeutic purposes that include ultrasound imaging and targeted drug delivery. We applied a Koopman linear quadratic regulator (KLQR) to control the nonlinear oscillations of a EMB through the applied acoustic field. We will use the acoustic echo emitted by the microbubble as an observable to estimate the microbubble states and control a spherical bubble using a Koopman Leuenberger observer framework. We will demonstrate the effectiveness of the modified KLQR controller in driving the Microbubble to follow arbitrarily prescribed radial oscillations and stabilize at nonequilibrium radii.

ALLAN M. AVILA

PULLBACK OPERATORS AND SPECTRAL INVARIANTS ON VECTOR BUNDLES

Koopman operator theory has become a powerful framework for the data-driven study of nonlinear dynamical systems. In this work, we generalize the classical Koopman framework by introducing a broader class of operators that act on sections of vector bundles over the system's state space. This perspective incorporates pullback operators that arise naturally in the dynamics of tensor fields, enabling a spectral analysis that extends beyond function spaces. As a key application, we show that Lyapunov exponents are encoded in the real part of the discrete spectrum of these generalized operators. Furthermore, we leverage the imaginary components of the spectrum to define a generalized Ruelle rotation number, applicable to systems of arbitrary dimension. Time permitting, we will present a numerical algorithm for computing the spectral properties of the infinitesimal generators associated with these operators.

TOMISLAV BAZINA

KOOPMAN-BASED PREDICTION OF HAND GRIP FORCE USING SEMG – DEMONSTRATION & OUTLOOK

Restoring hand function after stroke or musculoskeletal injury requires rehabilitation devices that can sense intention and deliver just-enough assistance. We demonstrate a real-time, ROS-based framework that merges surface electromyography (sEMG) with Koopman operator theory to estimate and forecast hand-grip force for adaptive, assist-as-needed control. A single forearm electrode pair is sufficient: after a one-time calibration of 20–30 seconds, the pipeline filters the sEMG, embeds it in a high-dimensional space, and learns a Koopman model that estimates grip force with high accuracy. The short-term prediction model is updated in real-time via Dynamic Mode Decomposition (DMD), enabling the robot to anticipate user effort and adapt its assistance accordingly. The workshop will include a live demonstration highlighting the system's accuracy and robustness in grasping tasks. Future work will (i) complete Koopman estimation and prediction system integration, (ii) unify the optimization and sensitivity pipeline for rapid generalization to additional grasps, and (iii) couple data-driven DMD with state-of-the-art sEMG decomposition to link hand dynamics to motor-unit activity—paving the way toward an integrated, novel rehabilitation glove.

PETAR BEVANDA

OPERATOR LEARNING FOR RELIABLE, EFFICIENT PREDICTION AND CONTROL

This talk presents a unified, nonparametric and data-driven framework for learning and exploiting finite-dimensional representations of infinite-dimensional evolution operators, with applications to both forecasting and optimal control in complex dynamical systems. Departing from minimal-coordinate models that often introduce nonlinear bottlenecks, we embrace linear operators on function spaces—trading finite dimensionality for continuum-level linearity. First, we introduce a nonparametric approach for operator-theoretic learning that decouples the regularity of the hypothesis from closure, yielding models that are both statistically efficient and computationally cheap to deploy. Our learning algorithms come with provable learning guarantees and consistency as data volume grows, achieving low forecast error under general conditions.

Building on these nonparametric foundations, we extend operator learning to control by developing a reproducing-kernel Hilbert space (RKHS) representation that naturally incorporates exogenous inputs or control actions. We integrate these models into two complementary, convex design frameworks: iterated model predictive control and operator-theoretic dynamic programming via kernel-based Hamilton–Jacobi–Bellman (kHJB) recursions. Notably, our kHJB method overcomes the curse of dimensionality and delivers globally optimal feedback laws without requiring any proprietary external solver.

ELA ĐIMOTI

STREAMING DMD AND DYNAMIC FORECASTING

The Koopman operator framework provides powerful tools for computational analysis of dynamical systems in data-driven scenarios. In particular, it is the theoretical bedrock for the (Extended) DMD, which is simply a numerical method for data snapshots-based extraction of spectral information of the Koopman operator associated with the dynamics.

In an online application, data snapshots are received in discrete time steps, possibly in batches, so that the (E)DMD has to be recomputed over a widening data window. At some point, the oldest data may gradually be forgotten. Further, to capture sudden changes in the dynamics and keep the forecasting skill in case of a black swan event, the oldest data are immediately discarded, which means that the data window may suddenly shrink and then keep widening again. All this requires fast updating of the decomposition, that matches the dynamics of the data windows widths, and restoring the forecasting capabilities.

We revisit the work of Hemati et al. and Zhang et al. on the online DMD and propose another approach that avoids the updates of the inverses via the Sherman-Morisson formula. Instead, the updates are based on orthogonal QR and LQ factorizations. Our numerical analysis and numerical examples demonstrate better numerical properties that result in better forecasting skill.

TITLES & ABSTRACTS

MASIH HASELI

KOOPMAN CONTROL FAMILY: EXTENDING KOOPMAN OPERATOR THEORY TO CONTROL

We introduce the Koopman Control Family (KCF), a Koopman-based axiomatic framework for representing general (not necessarily control-affine) discrete-time nonlinear control systems. The KCF includes all Koopman operators for autonomous (input-free) systems created by setting the control input to a constant value. We demonstrate that KCF can fully characterize control system behavior over an appropriate function space. Since KCF generally contains uncountably many elements, direct implementation is challenging. To address this, we parametrize KCF over the input set and show that its behavior is captured via a single linear operator on an augmented function space.

Given the importance of finite-dimensional models in control applications, we introduce a universal finite-dimensional form for the KCF, called the "input-state separable" model, using a generalized concept of subspace invariance. Notably, the input-state separable form includes the commonly used Koopman-based linear, bilinear, and switched linear models as special cases. When the subspace isn't invariant under the KCF, we provide best approximations in input-state separable form. Additionally, we characterize the accuracy of these approximated models via "invariance proximity," which measures worst-case prediction error across all functions in the finite-dimensional space. Finally, we discuss how our framework applies to data-driven modeling.

DAVID LIOVIĆ

BLOOD GLUCOSE LEVEL PREDICTION USING KOOPMAN OPERATOR THEORY

Accurate blood glucose prediction is crucial for effective diabetes management, especially for individuals with Type 1 diabetes who must make frequent decisions regarding insulin dosage, physical activity, and dietary intake. In this study, we introduce a personalized predictive algorithm based on Koopman Operator Theory, using data from continuous glucose monitoring (CGM) systems. The model relies solely on historical blood glucose levels as input, avoiding the need to include additional variables such as carbohydrate consumption, insulin administration, exercise, or stress levels. This simplified input structure enhances the model's practicality and usability in real-world settings. Despite its minimal input and early stage of development, the proposed approach demonstrated strong predictive performance across different time horizons. Validation on datasets from multiple individuals further confirmed the robustness and generalizability of the model. These findings suggest that Koopman-based modeling holds significant promise for personalized and low-effort blood glucose forecasting, providing a valuable tool to support real-time decision-making in diabetes self-care.

WILLIAM T. REDMAN

KOOPMAN LEARNING WITH EPISODIC MEMORY

Koopman operator theory has found significant success in learning models of complex, real-world dynamical systems, enabling prediction and control. The greater interpretability and lower computational costs of these models, compared to traditional machine learning methodologies, make Koopman learning an especially appealing approach. Despite this, little work has been performed on endowing Koopman learning with the ability to leverage its own failures. To address this, we equip Koopman methods – developed for predicting non-autonomous time-series – with an episodic memory mechanism, enabling global recall of (or attention to) periods in time where similar dynamics previously occurred. We find that a basic implementation of Koopman learning with episodic memory leads to significant improvements in prediction on synthetic and real-world data. Our framework has considerable potential for expansion, allowing for future advances, and opens exciting new directions for Koopman learning.

PAULA SANCHEZ

RESIDUAL ORDERING OF KOOPMAN SPECTRA FOR THE IDENTIFICATION OF TROPICAL SST FUNDAMENTAL MODES

El Niño–Southern Oscillation (ENSO) is a prominent driver of global climate variability, with significant impacts on ecosystems and societies. While existing empirical-dynamical forecasting methods, such as Linear Inverse Models (LIMs), are limited in capturing ENSO's inherent nonlinearity, Koopman operator theory offers a framework for analyzing such complex dynamics. Recent advancements in Koopman-based methods, such as DMD-based methods, have enabled exploration of nonlinear ENSO-related modes. However, they often suffer from challenges in robustness and interpretability. Specifically, k-EDMD algorithms tend to produce a large number of modes, complicating their physical relevance and reliability. In this study, we address these limitations by employing Colbrook's Residual EDMD (Res-EDMD) framework as a tool to classify and prioritize modes based on their residuals. Together with the application of pseudo-spectrum theory, this approach enables us to systematically identify robust and physically meaningful modes, distinguishing them from less reliable counterparts. Furthermore, leveraging the property that eigenfunctions of Koopman operators can generate higher-order harmonics through powers and multiplications, we introduce a methodology to detect fundamental modes and their associated harmonics. Applying this framework to tropical Pacific SST data, we demonstrate that k-EDMD, together with Res-EDMD, are capable of isolating fundamental modes of tropical SST dynamics. These fundamental modes provide insights into the system's physical evolution and facilitate the retrieval of meaningful dynamical information. By systematically identifying and interpreting the modes, we establish a pathway to overcome the limitations of conventional Koopman-based methods, thereby enhancing their applicability for studying and forecasting complex climatic systems like ENSO. This study underscores the potential of Res-EDMD to refine mode selection in Koopman spectral analysis, paving the way for robust, physically interpretable insights into tropical SST variability.